On the optimality of linear contracts to induce goal-congruent investment behaviour

Thomas Pfeiffer* and Louis Velthuis

Accounting and Management Control, University of Vienna, Brünner Str. 72, A-1210 Vienna, Austria

It has become increasingly popular in practice to implement incentive systems that create goal-congruent investment behaviour between central and divisional management. In the following paper, it is shown that only linear contracts enable goal-congruent investment decisions if central management does not have information about the investment project. This might cast a new light on why linear compensation schemes are often used in practice.

I. Introduction

A central issue in value-based management literature is the performance evaluation of managers who are responsible for making investment decisions for an organizational unit. For the purpose of incentive system design, it has become increasingly popular, in practice as well as in theory, to analyse and implement incentive systems that create goal-congruent investment behaviour between central and divisional management. That is, the manager should have an incentive to invest such that the net present value of the investment project is maximized.

The goal-congruence criterion, which has a long tradition in the field of investment decision making (see, for example, Solomons, 1965), has been recently re-examined in the context of incentive system design. Restricting his analysis to linear compensation rules, Reichelstein (1997, 2000) has shown that if managers are rewarded with the same proportion of residual income in each period, then goal congruence can be achieved provided that central and divisional management’s discount rates are the same and central management does not have any information about the investment project’s characteristics. This follows from the well-known conservation property of residual income, which has become the basis for modern value based management (see Preinreich, 1938; Feltham and Ohlson, 1995): that is, the net present value of any project equals the present value of residual incomes, if the initial investment is allocated across the different periods such that the present value of the cost allocation equals the initial investment. It seems natural to ask whether the focus on linear compensation rules was merely convenient from a technical perspective or whether a broader class of incentive systems would enable to induce goal-congruent behaviour. In the following study, we want to characterize the entire class of goal-congruent incentive systems if central management does not have any information about the investment project. In particular, the following questions are analysed:

(i) Are there other non-linear compensation rules based on cash flows or residual incomes that enable goal-congruent investment behaviour?
(ii) and, in the case of residual income, which allocation rule enables goal-congruent

*Corresponding author. E-mail: thomas.pfeiffer@univie.ac.at
investment behaviour. Must the allocation rule be complete? Or does there exist a non-complete allocation rule that enables goal-congruent investment behaviour in association with a non-linear compensation rule?

The purpose of this article is to characterize the entire class of goal-congruent incentive systems. The paper is organized as follows: Section II presents the basic model structure and basic definitions and Section III characterizes the class of goal-congruent incentive systems.

II. The Model

Like Rogerson (1997) and Reichelstein (1997, 2000), a firm is considered which must undertake an investment decision \( I \in \mathcal{I} \) at date 0 (\( I \geq 0 \)). The investment project \( P \) generates cash in- and outflows for \( T \) periods of (\( T \geq 2 \))

\[
P = (-I, c_1(I), \ldots, c_T(I))
\]

Thereby \( c_t(I) \) denotes the cash flow function at time \( t \). The investment project is one of all projects available to the firm and is selected at random from nature. By \( P \) the domain of all possible projects is denoted. Since the cash flows associated with the project are known only to the manager, the manager has to choose the level of investment for central management at the beginning of period 0.

Additionally, an accounting system associated with the investment project is considered. The accounting system tracks the \textit{ex post} realized investment level \( I \) at date 0 and the realized cash flow \( c_t(I) \) at date \( t \).

In addition to pure cash flows, the system also determines accruals by allocating the initial investment via an allocation rule \( a = (a_1, \ldots, a_T) \) across the different time periods. That is, if \( I \) is invested, then \( a_t I \) is allocated to period \( t \). An allocation rule \( a \) is called complete with respect to the interest rate \( r = (r_1, \ldots, r_T) \) if its present value is one

\[
\sum_{t=1}^{T} p_t a_t = 1
\]

As part of the incentive scheme, the agent is compensated according to an \textit{ex ante} chosen class of compensation rules \( \psi(\cdot, \cdot, \cdot) := (\psi_0(\cdot), \psi_1(\cdot), \ldots, \psi_T(\cdot)) \). The compensation rules are assumed to be continuously differentiable and strictly increasing in the chosen performance measure \( \Pi(\cdot) = (\Pi_0(\cdot), \Pi_1(\cdot), \ldots, \Pi_T(\cdot)) \) (formally, \( \Delta \psi_t(\Pi(I_1))/\Pi(I_t) > 0 \) for every \( t = 0, \ldots, T \)). The analysis is restricted to two commonly used classes of performance measures:

(i) currently realized cash flows

\[
\Pi^{CF}(I, P) = (-I, c_1(I), \ldots, c_T(I))
\]

or

(ii) residual income

\[
\Pi^{RI}(I, P) = (0, c_1(I) - a_1 I, \ldots, c_T(I) - a_T I)
\]

In summary, an incentive system \( S \) consists of the considered class of compensation rules \( \psi(\cdot, \cdot, \cdot) \), performance measures \( \Pi(\cdot) \) and of the allocation rule \( a \) in the case of residual income.

Central management would like the manager to invest \( I^*(P) \) into the project \( P \), so that the net present value of the project is maximized

\[
I(P) \in \arg \max \left\{ \text{NPV}(P, I) = \sum_{t=1}^{T} p_t c_t(I) - I \mid I \in \mathcal{I} \right\}
\]

given the firm’s cost of capital \( r \). Recent literature on incentive system design has pointed out that managers tend to be relatively ‘impatient’ and therefore more short-term focused than the firm (Rogerson, 1997; Reichelstein, 1997, 2000; Pfeiffer, 2000; Dutta and Reichelstein, 2004). Consistent with literature, this fact is captured by assuming that the manager discounts his future payoffs with other discount factors \( q = (q_1, \ldots, q_T) \) than central management. \( W^S(P, I) \) denotes the present value of the manager’s future compensation

\[
W^S(P, I) = \sum_{t=0}^{T} q_t \psi_t(I_1(I))
\]

given the incentive system \( S \).

In the following, the focus is on goal-congruent incentive systems. It is stated that an incentive system \( S = (\psi(\cdot, \cdot, \cdot), a) \) consisting of the compensation rules \( \psi(\cdot, \cdot, \cdot) := (\psi_0(\cdot), \ldots, \psi_T(\cdot)) \), performance measures \( \Pi(\cdot) = (\Pi_0(\cdot), \ldots, \Pi_T(\cdot)) \) and the allocation rule \( a = (a_1, \ldots, a_T) \) leads to goal-congruent investment decisions if and only if

\[
\arg \max \{\text{NPV}(P, I) \mid I \in \mathcal{I} \}
= \arg \max \{W^S(P, I) \mid I \in \mathcal{I} \} \quad \forall P \in \mathcal{P}
\]

is satisfied. Condition A states that the manager should select the investment the headquarters would have selected if it had perfect information about the project \( P \) for all investment projects considered. But Condition A does not require that the present value of the manager’s future compensation is equivalent to the net present value of the project. For example, every strictly increasing function of the net present
value rule also enables goal-congruent investment decisions.

III. Constructing Goal-Congruent Incentive Systems

In this section, the class of goal-congruent incentive systems for general investment projects \((-b, c_1(I), \ldots, c_T(I))\) is analyzed without restricting the cash flows to, say, strictly increasing and concave functions. Proposition 1 provides an if and only if characterization of the class of goal-congruent incentive schemes based on (i) cash flows and on (ii) residual incomes (a proof of Proposition 1 is given in the Appendix).

Proposition 1 If the manager’s discount factors are known to central management, then the class of goal-congruent incentive schemes based on (i) cash flows and on (ii) residual incomes (a proof of Proposition 1 is given in the Appendix).

(i) \(S = (\psi(\cdot), \Pi^\text{CF}(\cdot))\) based on cash flows enable goal-congruent investment decisions if and only if the compensation schemes are linear

\[
\psi_t(\Pi_t^\text{CF}(P, I)) = k\frac{P_t}{q_t}\Pi_t^\text{CF}(P, I) + w_t
\]

\((k > 0, w_t \in \mathbb{R}, t = 0, \ldots, T)\)

where \(k p_t/q_t\) denotes the bonus coefficient and \(w_t\) the fixed amount at time \(t\).

(ii) \(S = (\psi(\cdot), \Pi^\text{Inc}(\cdot), a)\) based on residual incomes enable goal-congruent investment if and only if compensation schemes are linear

\[
\psi_t(\Pi_t^\text{Inc}(P, I)) = k\frac{P_t}{q_t}\Pi_t^\text{Inc}(P, I) + w_t
\]

\((k > 0, w_t \in \mathbb{R}, t = 1, \ldots, T)\)

and the allocation rule is complete with respect to central management’s capital cost rates

\[
\sum_{t=1}^{T} p_t a_t = 1
\]

Proposition 1 provides an if and only if characterization of how goal-congruent incentive schemes have to be constructed based (i) on pure cash flows and (ii) on residual incomes. Proposition 1 shows that for both classes of performance measures, the structure of the compensation scheme must be linear \(\psi_t(\Pi_t) = k(p_t/q_t)\Pi_t\) consisting of a bonus coefficient \(k(p_t/q_t)\) and a fixed amount \(w_t\). Because Proposition 1 is an if and only if characterization, there can be no other non-linear compensation scheme and no other non-complete allocation rule to achieve goal congruence between central and divisional management. Thereby, the structure of the compensation scheme and the allocation rule are both simultaneously endogenized (see the second part of the proof of (i) and (ii)). Starting from linear compensation schemes and complete allocation rules Reichelstein (1997) shows that the bonus coefficients must remain constant over the periods of time for residual income, if central and divisional management have identical cost rates. Thus, Proposition 1 shows that the assumption about linear compensation rules made by Reichelstein is not a restriction, because this is the only class that enables goal-congruent investment behaviour. The fact that goal-congruent investment behaviour can be induced only via linear incentive contracts might cast a new light on why companies in the real world use linear compensation schemes.

Under both classes of performance measures, the bonus coefficients are constructed identically. If headquarters and divisional management have identical discount factors, then the bonus coefficients must remain constant over all periods. Otherwise, the bonus coefficient must be constructed to smooth intertemporal distortions resulting from the different discount factors. Although Condition (A) does not require that the incentive system must be a linear transformation of the net present value of the project, it is important to note that under both incentive schemes the net present value of the compensation calculated with the manager’s discount rates must be a linear transformation of the net present value of the project calculated with headquarters’ capital cost rates

\[
W^*(P, I) = k NPV(P, I) + w \quad \forall P \in \mathcal{P}
\]

It is interesting to note that a separation between the allocation and the compensation rule applies by incorporating the manager’s discount rates. This means, that the (other) discount rates of the manager can only be incorporated into the incentive system via the bonus coefficients and not via the allocation rule. Furthermore, the allocation rule must be complete with respect to headquarters’ capital cost rates. Proposition 1 shows that it is not possible to construct any other goal-congruent incentive system based on a non-complete allocation rule. Thus, the result provides a rationale for the concept of complete allocation rules, which are typically used in accounting.

References


Pfeiffer, Th. (2000) Good and bad news for the implementation of shareholder-value concepts in decentralized organizations, Schmalenbach Business Review, 68–90.
Appendix

Proof (i) Part 1: First, it is obvious that the cash flow-based incentive scheme enables goal congruence

\[ \arg \max \left\{ W_{SCr}(P, I) = \sum_{t=1}^{T} q_t \left( \frac{p_t}{q_t} c_t(I) + w_t \right) - k I | I \in \mathcal{I} \right\} \]

\[ = k \arg \max \left\{ \sum_{t=1}^{T} p_t c_t(I) - I | I \in \mathcal{I} \right\} + \sum_{t=0}^{T} q_t w_t \]

for all investment projects \( P \in \mathcal{P} \). This completes the first part of (i).

(ii) Part 2: To show the second part of the proof, it is assumed that another incentive scheme exists that enables goal congruence between divisional management and headquarters for all investment projects. In particular, the incentive scheme must also exhibit goal congruence for all differentiable investment projects, for which the optimal investment level of the headquarters is given by its first order condition

\[ 0 = \frac{\partial NPV(P, I)}{\partial I} = \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial c_t} - 1 \]

The first order condition of the divisional manager must also hold at the optimal investment level of headquarters

\[ 0 = \frac{\partial W_{SCr}(P, I)}{\partial I} = \sum_{t=1}^{T} q_t \frac{\partial \psi_t(c_t(I))}{\partial c_t} \frac{\partial c_t(I)}{\partial I} - \frac{\partial \psi_0(I)}{\partial I} \]

Multiplying the first order condition of divisional management with \( 1/k > 0 \) and applying the first order condition of the headquarters yields

\[ \frac{1}{k} \left( \sum_{t=1}^{T} q_t \frac{\partial \psi_t(c_t(I))}{\partial c_t} \frac{\partial c_t(I)}{\partial I} - \frac{\partial \psi_0(I)}{\partial I} \right) = \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial c_t} - 1 \]

This leads to

\[ 0 = \sum_{t=1}^{T} \frac{\partial \psi_t(c_t(I))}{\partial c_t} \left( \frac{1}{k} \frac{\partial \psi_t(c_t(I))}{\partial c_t} - q_t \right) - \left( \frac{1}{k} \frac{\partial \psi_0(I)}{\partial I} \right) \]

This is a polynomial of degree one in \( (y_0, \ldots, y_T) := (-1, \frac{\partial \psi_t(c_t(I))}{\partial I}, \ldots, \frac{\partial \psi_0(I)}{\partial I}) \), which must be identical to zero for all \( (y_0, \ldots, y_T) \) (because, by assumption, headquarters neither knows the optimal investment level nor the cash flow functions). According to the fundamental theorem of algebra this is true only if all coefficients are set equal to zero. This leads to

\[ q_t \frac{1}{k} \frac{\partial \psi_t(c_t(I))}{\partial c_t} = p_t \quad (\forall I \in \mathcal{I}, t = 1, \ldots, T) \]

and

\[ \frac{1}{k} \frac{\partial \psi_0(I)}{\partial I} = 1 \quad (\forall I \in \mathcal{I}) \]

which must be satisfied for all possible investment levels \( I (I \in \mathcal{I}) \). The cash flow function \( c_t(I) \) varies with the investment level \( I \). The cash flow function \( c_t(I) \), and the compensation scheme \( \psi_t(I) \), which is increasing in the cash flow \( c_t(I) \), also varies with the investment level \( I \). The only class of all differentiable functions where the first derivative is constant over an interval is the class of linear functions. Hence, to assure that the first equation is satisfied, the compensation scheme must be linear

\[ \psi_t(c_t(I)) = k \frac{p_t}{q_t} c_t(I) + w_t \quad (t = 1, \ldots, T) \]

and

\[ \psi_0(I) = k I + w_0 \]

This completes the proof of (i).

(iii) Part 1: First, it is shown that the income based incentive scheme of part (ii) enables goal congruence

\[ \arg \max \left\{ W_{Sinc}(P, I) = \sum_{t=1}^{T} q_t \left( k \frac{p_t}{q_t} (c_t(I) - a_t(I)) + w_t \right) | I \in \mathcal{I} \right\} \]

\[ = \arg \max \left\{ k \left( \sum_{t=1}^{T} p_t (c_t(I) - a_t(I)) \right) | I \in \mathcal{I} \right\} + \sum_{t=1}^{T} q_t w_t \]

\[ = k \arg \max \left\{ \sum_{t=1}^{T} p_t c_t(I) - I | I \in \mathcal{I} \right\} + \sum_{t=1}^{T} q_t w_t \]

for any investment project \( P \in \mathcal{P} \). This completes the first part of (ii).

(ii) Part 2: To show the second part of the proof, it is assumed that another incentive scheme exists...
which exhibits goal congruence for all differentiable investment projects, for which the optimal investment level of the headquarters is given by its first-order condition

$$0 = \frac{\partial NPV(P, I)}{\partial I} = \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial I} - 1$$

To induce goal congruence, the first-order condition of the divisional manager must also hold at the optimal investment level of headquarters

$$0 = \frac{\partial W^{\text{Inc}}(P, I)}{\partial I}$$

$$= \sum_{t=1}^{T} q_t \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} \frac{\partial \Pi_{t}^{\text{Inc}}(I)}{\partial I}$$

$$= \sum_{t=1}^{T} q_t \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} \left( \frac{\partial c_t(I)}{\partial I} - a_t \right)$$

Multiplying the first-order condition of divisional management with $1/k > 0$ and applying the first-order condition of the headquarters yields

$$\frac{1}{k} \sum_{t=1}^{T} q_t \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} \left( \frac{\partial c_t(I)}{\partial I} - a_t \right)$$

$$= \sum_{t=1}^{T} p_t \frac{\partial c_t(I)}{\partial I} - 1$$

This leads directly to

$$0 = \sum_{t=1}^{T} \frac{\partial c_t(I)}{\partial c_t} \left( q_t \frac{1}{k} \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} - p_t \right)$$

$$+ \left( 1 - \sum_{t=1}^{T} q_t \frac{1}{k} \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} a_t \right)$$

This is a polynomial of degree one in $(y_0, \ldots, y_T) = (-1, \frac{\partial c_1(I)}{\partial I}, \ldots, \frac{\partial c_T(I)}{\partial I})$, which must be identical to zero for all $(y_0, \ldots, y_T)$ (because, by assumption, headquarters neither knows the optimal investment level nor the cash-flow functions). According to the fundamental theorem of algebra this is true only if all coefficients are set equal to zero. This leads to

$$q_t \frac{1}{k} \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} = p_t \quad (\forall I \in \mathcal{I})$$

and

$$\sum_{t=1}^{T} q_t \frac{1}{k} \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} a_t = 1 \quad (\forall I \in \mathcal{I})$$

which must be satisfied for all possible investment levels $I (I \in \mathcal{I})$. The cash flow function $c_t(I)$ increases with the investment level $I$. The performance measure $\Pi_{t}^{\text{Inc}}(I) = c_t(I) - a_t I$, and the compensation scheme $\psi_t(I)$, which is increasing in the performance measure $\Pi_{t}^{\text{Inc}}(I)$, also varies with the investment level $I$. The only class of all differentiable functions where the first derivative is constant over an interval is the class of linear functions. Hence, to assure that the first equation is satisfied, the compensation scheme must be linear

$$\psi_t(\Pi_{t}^{\text{Inc}}(I)) = k \frac{p_t}{q_t} \Pi_{t}^{\text{Inc}}(I) + w_t$$

Thus, the second equation reduces to

$$\sum_{t=1}^{T} q_t \frac{1}{k} \frac{\partial \psi_t(\Pi_{t}^{\text{Inc}}(I))}{\partial \Pi_{t}^{\text{Inc}}(I)} a_t$$

$$= \sum_{t=1}^{T} q_t \frac{1}{k} p_t a_t = \sum_{t=1}^{T} p_t a_t = 1$$

This completes the proof of (ii).